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School Bus Routing Problem with Fuzzy Walking Distance

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***Abstract*—The School Bus Routing Problem is a type of Vehicle Routing Problem that aims to optimize the planning of bus routes for a school. This problem has received increased interest in the last decade. One of the aspects that stand out most in the progress of optimization problems is making them as close to reality as possible. In this sense, fuzzy optimization is a suitable way to do this by considering certain levels of uncertainty. Although the fuzzy approach has been applied to the Vehicle Routing Problem, it has not been so with the School Bus Routing Problem. Therefore, the objective of this paper is to introduce a fuzzy model for the School Bus Routing Problem, particularly with the maximum student walking distance as a fuzzy element. This fuzzy version of the School Bus Routing Problem allows obtaining a set of solutions with different trade-offs between cost and relaxation of the original conditions. The results obtained in 31 instances by using the parametric approach are analyzed, taking into account three characteristics of the problem: number of bus stops, number of students, and walking distance. It is shown that the introduced fuzzy version is useful for decision-makers by providing relaxed alternative solutions with significant cost savings.**

***Index Terms*—Fuzzy optimization, parametric approach, School Bus Routing Problem.**

1. Introduction

THE SCHOOL Bus Routing Problem (SBRP) aims to create, in an optimal way, a set of routes for the transportation of students to their schools from different locations [1]. This problem has been extensively studied since the publication of [2].

The SBRP is considered a type of Vehicle Routing Problem (VRP) [3]. From the point of view of optimization, in the creation of the routes, several objectives may be taken into account, e.g., minimizing the total distance traveled [4] or minimizing the number of buses to be used [5]. Also, a set of restrictions must be met, e.g., the capacity of buses [6] or the entry and exit times to schools [7]. The SBRP has been applied in different settings, e.g., in [8], workers are transported to their places of work instead of students.

On the other hand, the management of uncertainty is one of the ways to obtain models and solutions to situations more similar to reality. Two outstanding techniques for uncertainty management are stochastic approximations [9] (where some of the elements of the problem present a random behavior) and fuzzy optimization [10], where some elements involve certain levels of subjectivity or ambiguity.

In the case of fuzzy optimization, there are several studies on its application in VRP, its variants, and in other optimization routing problems. For example, in [11], several models were developed for variants of the VRP with imprecise travel times modeled as fuzzy triangular numbers. The authors of [12] present a fuzzy multi-objective optimization problem to model single frequency routes bus timetabling and solve the model by a kind of genetic algorithm. In [13], a model for the Truck and Trailer Routing Problem (TTRP) with imprecise capacity restrictions and fuzzy treatment was proposed. More recently, [14] presented an integrated production inventory routing problem (IRP) with fuzzy approximation in the demand of retailers. Likewise, in [15], the Fuzzy Green Vehicle Routing Problem (FGVRP) is considered for the design of a supply chain, where customer demands are considered fuzzy. Finally, the authors of [16] introduced a Fuzzy Electric Vehicle Routing Problem with time windows and recharging stations (FEVRPTW), where fuzzy numbers were used to treat the uncertainty of service times, battery energy consumption and travel times.

As it has been shown, in these examples of VRP models and their variants, various aspects have been considered fuzzy, such as travel time, service times, customer demand, or vehicle capacity. However, to the best of our knowledge, similar techniques have not been applied to SBRP. Particularly in the SBRP, one of the elements that can be treated as fuzzy is the student's walking distance to reach the bus stops. The distance has been treated as a fuzzy element in other optimization problems, such as the MCLP [17] from the point of view of coverage, with interesting results.

When modeling the student walking distance as a fuzzy element, the decision-maker interested in the solution of an SBRP would have the opportunity to obtain a diverse set of solutions with different trade-offs between the relaxation of the original constraints and the optimization objectives. With this set of solutions, the decision-maker could select among them the one that most satisfies a particular interest. For example, assume an SBRP instance where the objective is to minimize the total distance traveled by buses and the student's walking distance is restricted to 300 meters. In this situation, the best solution, which is the sum of all routes, may have a cost of 15 kilometers. However, if the walking distance is considered a fuzzy constraint, it may be treated in a relaxed way. For example, if the walking distance is increased to 350 meters, the best solution may decrease to 13.5 kilometers, while if the limit is increased to 400 meters, the best solution may be 13 kilometers. Hence, the decision-maker would have three options.

In the previous example, a linguistic value can be used to classify the distance between students and stops. This linguistic value can be "admissible" or "not admissible." When the student's

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walking distance is treated as fuzzy, this distance can be "admissible" with different degrees of satisfaction in [0,1].

Taking these antecedents into account, the objective of this work is to introduce a fuzzy model of the SBRP and thus to demonstrate its impact on solving this type of problem. In particular, in the proposal, the restriction that limits the maximum student's walking distance becomes fuzzy.

The rest of the document is organized as follows. Section 2 addresses the general characteristics of the SBRP and a study of the literature on the subject. In Section 3, the mathematical model and its fuzzy approach for the SBRP are presented. Section 4 presents and discusses the experimental results. Finally, the conclusions and future work are presented in Section 5.

2. School Bus Routing Problem

The School Bus Routing Problem (SBRP) is defined to ensure an optimal transportation policy for the students of a school or school district. It was identified as a problem in [2] more than 60 years ago. However, it is not until the last decades that contributions to the modeling and solution of this problem have increased.

According to [18] and [19], the SBRP can be divided into five sub-problems: 1) Preparation of data, 2) Selection of bus stops, 3) Generation of routes, 4) Route calendar, and 5) Adjustment to school bell time. On the other hand, [20] suggests that the preparation of the data (1) can be part of each sub-problem, and therefore treats it as such and not as a sub-problem. Likewise, it describes a new sub-problem proposed in [21] called strategic transportation policies (6).

There are several approaches for modeling and solving SBRP (see [19] and [20]). In this way, the SBRP sub-problems can be classified according to:

1. Number of schools: one or multiple schools.
2. Service environment: urban or rural.
3. Load type: distinguishes whether a bus can carry students to more than one school. Mixed loads generally refer to allowing a bus to carry students from multiple schools.
4. Fleet mix: identifies whether the buses under consideration have the same capacity (homogeneous) or varying capacities (heterogeneous).
5. Objective: generally, it is the minimization of one or more of the following aspects:
    a. Number of buses (N).
    b. Total bus traveled distance or time (TBD).
    c. The total student riding distance or time (TSB).
    d. Total student walking distance (SWD).
    e. Maximum route length (MRL).
6. Constraints: normally, in each SBRP model, it is proposed to meet one or more of the following conditions:
    a. Vehicle capacity (C).
    b. Maximum riding time (MRT).
    c. School Time Window (TW).
    d. Maximum walking time or distance (MWT).

From the solution point of view, multiple approaches have been used, including some exact, heuristic, and metaheuristic algorithms. Before the study presented in [19], only a few solutions based on metaheuristics could be found. However in the last 10 years, the interest in SBRP has substantially increased and especially the use of metaheuristic algorithms [20], such as Genetic Algorithms (e.g. [22], [23], [24]), Ant Colonies (e.g. [25], [26], [27]), Simulated Annealing (e.g. [28]), Tabu Search (e.g. [29], [30]) and GRASP (e.g. [31], [32], [33]). The use of metaheuristics in the SBRP solution is mainly due to the increase in size and complexity of the instances to be solved. Furthermore, these algorithms have been shown to provide good solutions to other combinatorial optimization problems similar to the SBRP.

3. Mathematical Model

In [34], a metaheuristic solution for SBRP with bus stops selection and homogeneous fleet was presented. The objective was to minimize the total distance traveled by the buses, with constraints related to the bus capacity and the students walking distance.

The fuzzy model presented here is a fuzzy extension of the model presented in [34]. The main difference is that the model presented here allows a relaxation of the student's walking distance. In general terms, the new model is described in terms of the variables used in [34].

3.1 Variables

Input variables

c: Capacity of each bus.
b: Number of buses.
d: Maximum students walking distance.
P: Set of possible stops.
E: Set of students.
C^p: A set of vectors with coordinate pairs of the possible stops.
C^e: A set of vectors with pairs of coordinates of the house of each student.

Auxiliary Variables

C_{ij}: Cost matrix between each pair of stops (i, j).
D: A distance that indicates the cost between a pair of stops or between a student and a bus stop. Euclidian distance is used.
${c^p}_i$: Are the coordinates of the stop located on the i index of C^p.

$$C_{ij} = \begin{cases} D\left({c^p}_i, {c^p}_j\right), & i \neq j \\ 0, & i = j \end{cases} \quad \forall i, \forall j \in P \tag{1}$$

S_{pe}: Binary matrix. 1 if the student j can reach the stop i, and 0 otherwise.
${c^e}_j$: Are the coordinates of the student located on the j index of C^e.
p_0: This is the first element of P, which means the location of the school.
$\leq^f$: Indicates the imprecision of the constraint (2).

$$S_{pe} = \begin{cases} 1, D\left({c^p}_i, {c^e}_j\right) \leq^f d \\ 0, D\left({c^p}_i, {c^e}_j\right) >^f d \end{cases} j \in E, i \in P - \{p_0\} \tag{2}$$

Decision Variables

R_{km}: Indicates the stop that is visited by the bus k in the order m.
Z_e: Indicates the stop where the student e is picked up

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3.2 Objective Function and Constrains

Objective Function

$$\mathrm{Min} \sum_{k=1}^{b} \sum_{m=1}^{|P|-1} C_{[R_{km}][R_{km+1}]} \quad (3)$$

Constrains

$$|\{R_{km} | R_{km} = p\}| \leq 1$$
$$\forall p \in P - \{p_0\}, \forall k \in \{1, \dots, b\}, \forall m \in \{1, \dots, |P|\} \quad (4)$$
$$\{(e,p) | Z_e = p\} \subseteq \{(e,p) | S_{pe} = 1\}$$
$$\forall e \in E, \forall p \in P \quad (5)$$
$$|\{e | \exists m \; R_{km} = Z_e\}| \leq c$$
$$m \in \{0, \dots, |P|\}, \forall k \in \{1, \dots, b\}, \forall e \in E \quad (6)$$
$$|\{R_{km} | Z_e = R_{km}\}| = 1$$
$$\forall e \in E \quad (7)$$

The objective function, equation (3), minimizes the total distance traveled by the entire bus fleet. Equations (4), (5), (6) and (7) represent the restrictions that must be met for the solution to be feasible. Equation (4) guarantees that each stop is visited at most once, except for stop p_0, which represents the school, the final destination of all buses. Equation (5) ensures that each student can reach their assigned bus stop. Equation (6) takes into account that the capacity of each bus is not exceeded on the route. And, finally, with Equation (7), it is guaranteed that each stop to which at least one student is assigned is visited by a bus.

Concerning the model introduced in [34], the difference is that the model in [34] wasn't fuzzy. Particularly, the fuzzy nature of the model presented here is related to equation (2), where the fuzzy operators ($>^f, \leq^f$) are used, replacing the crisp operators ($>, \leq$) used in [34].

3.3 Fuzzy model for SBRP

As can be seen in the previous model, restriction (5) depends on the value of the auxiliary variable S_{pe}. This is the focus of the proposed fuzzy model. The new way of posing this constraint implies that the feasibility of a student reaching a stop becomes fuzzy (i.e., not crisp) and therefore has different degrees of membership.

Taking this element into account, if the maximum walking distance for students is 200 meters, then a student that walks 190 meters satisfies it with a grade of 1. On the other hand, if a student walks 210 meters, the degree of membership (degree of satisfaction of this constraint) may be less than 1, but higher than if the stop is 250 meters away. Conversely, a stop that is located 500 meters away could be considered unreachable. All these values (e.g., 200, 500) will depend on the admissible conditions and the allowed tolerance. These values will imply that solutions will have different degrees of compliance with the restrictions. From the decision-making point of view, this relaxation allows a small increase in the distance that the students could walk to find a relaxed solution with a better value of the objective function (reduced cost).

To model this situation, it is necessary to define a tolerance H, which determines the maximum admissible distance that a student could walk. Fig. 1 shows the function to measure the degree of compliance with the restriction taking into account the distance d and the tolerance H.

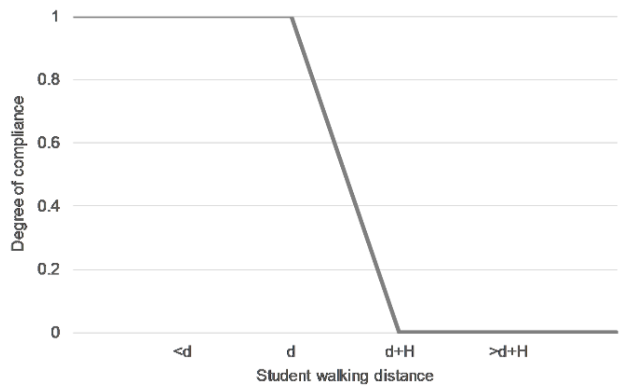

Fig. 1. Membership function of the compliance of the constraint associated with the student walking distance.

To understand this function, having d as the original maximum student walking distance and H as the maximum admissible tolerance, a student located at any distance less or equal to d has a degree of compliance of 1. On the other hand, if the student is located at any distance between d and d+H, it has a degree of compliance in the interval [0,1]. Finally, if the student location is at any distance greater than d+H to a stop, the degree of compliance is 0, and then it is assumed that the student can't reach the stop.

This is a linear function, and the parametric approximation method [10] can be applied based on the principles of parametric linear programming and the concept of alpha-cuts [10]. The concept of alpha-cut applied to this case implies that different sets of feasible solutions are associated with a particular value of alpha, i.e., those solutions with a degree of feasibility (the accomplishment of the original conditions) equal or greater than alpha. Consequently, with smaller values of alpha, some relaxed solutions are considered feasible. Then, instead of using the previous expression (2) now the expression (8) is used to obtain S_{pe}:

$$S_{pe} = \begin{cases} 1, D({c^p}_i, {c^e}_j) \leq d + H(1-\alpha) \\ 0, D({c^p}_i, {c^e}_j) > d + H(1-\alpha) \end{cases} \quad (8)$$

With this change, when $\alpha = 1$ the problem remains crisp, and then students can only reach those bus stops that are at the maximum original walking distance, i.e., it is the most restrictive case. On the other hand, when $\alpha = 0$, students are allowed to reach those bus stops that are at the maximum original distance plus the maximum tolerance, i.e., it is the greatest relaxation.

3.4 Solution approach of the fuzzy SBRP model

To illustrate the solution of the new fuzzy SBRP model, we use the parametric approach. In Fig. 2, a diagram gives a general picture of this approach, where a fuzzy problem ($P_\sim$) is transformed into a set of crisp problems ($\cup_\propto P_\propto$), where each problem corresponds to a different value of alpha ($\propto$). A solution is found for each of these crisp problems ($S_\propto$) and finally, the set of these solutions ($\cup_\propto S_\propto$), each associated with each alpha value, forms the solution of the original fuzzy problem ($S_\sim$).

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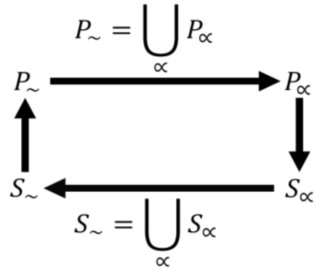

Fig. 2. Descriptive diagram of the parametric approach [35]. The cycle starts in the upper left corner.

To solve each crisp instance, any of the available solution methods for SBRP may be used. Here, we use the metaheuristic method presented in [34] to solve each crisp problem. This method consists of a metaheuristic that combines two heuristic algorithms for the construction of the initial solution with a local search strategy that has a probabilistic-based selection mechanism for mutation operators.

The first heuristic is in charge of assigning each student to a bus stop, while the second is in charge of building the routes for each bus that start from the school and go through each stop where there is at least one assigned student, to finally go back to school. This last heuristic has a greedy approach.

Four mutation operators are available for the selection mechanism for local search: Swap, Two-opt, Section swap, and Reorder.

In [34] this method was validated in the solution of the test instances previously used in [31]. These previous results demonstrate the efficacy of this metaheuristic approach that can reach the best-reported solutions in several instances and good solutions in all cases.

4. Results and Discussion

To validate the proposed fuzzy model for SBRP, as well as the algorithms used in its solution, 31 instances of problems were selected from the set studied in [34]. The characteristics of these instances can be seen in Table 1. In the case of the column Maximum Walking Distance for students, the unit for the values is “unit” in a Euclidean plain because the instances are artificial and a generic approach was applied. In a practical situation, each distance may be expressed in terms of meters or minutes.

Following what was stated in [36], these instances can be grouped according to the number of bus stops, a key element of SBRP. Particularly, there are six instances with 5, 10, and 40 bus stops, eight instances with 20 bus stops, and five instances with 80 bus stops.

Taking into account that the instances in Table 1 are the original crisp instances, each one induces an instance of the proposed fuzzy model. For the creation of these fuzzy instances, the tolerance was set to 20% of the original walking distance, i.e., H = 0.2 * d. Following the parametric approach, five values of alpha allow different relaxation for each fuzzy instance, thus allowing different degrees of membership to the crisp case, $\alpha \in \{0, 0.25, 0.5, 0.75, \text{and } 1\}$. By combining these alpha values, the solution of the 31 fuzzy instances derived in the solution of 31 × 5 = 155 crisp instances.

TABLE 1
CHARACTERISTICS OF THE 31 INSTANCES

Id	Instance	Stops count	Students count	Maximum walking distance for students	Capacity of the buses
1	Inst6	5	25	20	50
2	Inst9	5	50	5	25
3	Inst11	5	50	10	25
4	Inst15	5	50	40	25
5	Inst21	5	100	20	25
6	Inst24	5	100	40	50
7	Inst27	10	50	10	25
8	Inst32	10	50	40	50
9	Inst33	10	100	5	25
10	Inst37	10	100	20	25
11	Inst40	10	100	40	50
12	Inst42	10	200	5	50
13	Inst54	20	100	20	50
14	Inst55	20	100	40	25
15	Inst56	20	100	40	50
16	Inst57	20	200	5	25
17	Inst60	20	200	10	50
18	Inst64	20	200	40	50
19	Inst70	20	400	20	50
20	Inst72	20	400	40	50
21	Inst78	40	200	20	50
22	Inst79	40	200	40	25
23	Inst80	40	200	40	50
24	Inst84	40	400	10	50
25	Inst90	40	800	5	50
26	Inst95	40	800	40	25
27	Inst97	80	400	5	25
28	Inst99	80	400	10	25
29	Inst102	80	400	20	50
30	Inst108	80	800	10	50
31	Inst112	80	800	40	50

Table 2 shows the best solution value obtained for each instance from 30 executions of the metaheuristics described in [34] with 10,000 evaluations of the objective function. Each row constitutes the fuzzy solution for each instance of the fuzzy problem.

From these results, it can be observed that in 6 instances were obtained five different solutions, one for each level of relaxation or alpha-cut. In other instances, a reduced number of different solutions are obtained because some more relaxed cases do not imply an improvement in the objective function. From the point of view of a decision-maker, there is no rationale to allow a greater relaxation of a constraint if it does not imply an improvement in the cost. Thus, the solutions of interest are highlighted in bold in Table 2 because they are relevant to the decision-maker. For example, in Inst6, only two values are highlighted in bold, alpha=1 and alpha=0. That means that with the other alpha-cuts (0.75, 0.5, 0.25), the solution is not better than the solution with alpha=1; thus, these solutions should not be taken into account.

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TABLE 2
RESULTS OBTAINED FOR THE 31 FUZZY INSTANCES

Instance	Alpha				
	1	0.75	0.5	0.25	0
Inst6	**110.058**	110.058	110.058	110.058	**97.773**
Inst9	**286.681**	286.681	286.681	286.681	286.681
Inst11	**193.551**	193.551	193.551	**175.911**	175.911
Inst15	**13.794**	**11.072**	**9.069**	9.069	9.069
Inst21	**159.909**	159.909	159.909	159.909	159.909
Inst24	**39.807**	39.807	**33.036**	**24.167**	**12.033**
Inst27	**266.064**	266.064	266.064	266.064	**254.194**
Inst32	**56.882**	**32.797**	32.797	**25.791**	25.791
Inst33	**403.178**	403.178	403.178	403.178	403.178
Inst37	**220.359**	220.359	**198.066**	**173.156**	173.156
Inst40	**38.360**	38.360	**36.690**	**24.545**	**21.231**
Inst42	**506.060**	506.060	506.060	506.060	506.060
Inst54	**216.126**	**176.256**	**156.128**	**148.115**	148.115
Inst55	**57.540**	**43.106**	**35.732**	**31.920**	**18.805**
Inst56	**28.327**	**18.872**	18.872	**8.346**	**4.172**
Inst57	**932.522**	**928.684**	928.684	928.684	928.684
Inst60	**488.853**	**488.622**	**488.232**	**477.221**	**468.663**
Inst64	**55.197**	55.197	**32.686**	**30.206**	**28.894**
Inst70	**340.791**	**330.395**	330.395	330.395	**328.709**
Inst72	**95.540**	**79.565**	**63.200**	63.200	63.200
Inst78	**354.306**	**333.627**	**326.656**	**284.662**	**247.585**
Inst79	**95.226**	**89.373**	**63.636**	63.636	63.636
Inst80	**72.364**	**50.971**	**34.724**	**24.971**	**21.628**
Inst84	**840.655**	840.655	**800.118**	800.118	800.118
Inst90	**1376.405**	1376.405	1376.405	**1364.521**	1364.521
Inst95	**418.702**	**404.616**	**393.256**	393.256	393.256
Inst97	**1686.818**	1686.818	1686.818	1686.818	1686.818
Inst99	**1401.222**	1401.222	**1354.376**	1354.376	**1300.822**
Inst102	**566.535**	**513.113**	**474.049**	**439.875**	**427.293**
Inst108	**1459.557**	**1393.490**	1393.490	1393.490	**1368.123**
Inst112	**142.167**	**131.811**	**122.653**	**108.354**	**103.563**

Of the instances with five interesting solutions, instances Inst55 and Inst80 stand out because their greatest possible relaxation ($\alpha = 0$) implies a cost reduction of almost 70%. Fig. 3 illustrates the interesting trade-off between cost (y-axis) and relaxation (alpha value in x-axis) of the fuzzy solution of the instance Inst80 with five solutions of interest. In this case, the solution value with alpha 0 was 21.6284, which represents a 0.3 fraction of the solution value with alpha 1, 72.3635. Therefore, 70% of cost reduction can be obtained in this instance when the maximum relaxation is used.

In five of the instances, four solutions of interest were obtained; in 9 instances, three possible solutions; and six instances offer two different solutions. On the other hand, only in five instances (Inst9, Inst21, Inst33, Inst42, Inst97), the relaxation does not contribute any improvement to the original solution, i.e., the most relaxed solution ($\alpha = 0$) does not imply any improvement. The fact that a better solution than the original one can't be found in these instances is mainly because the change in the constraints of the instance does not allow a significant change in the alternative stops that are available for each student.

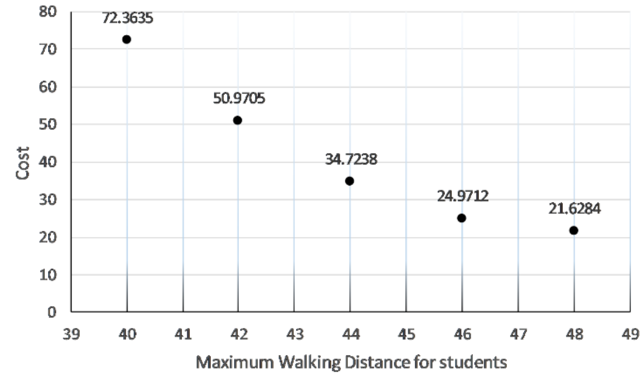

Fig. 3. Instance Inst80 solution in all membership values.

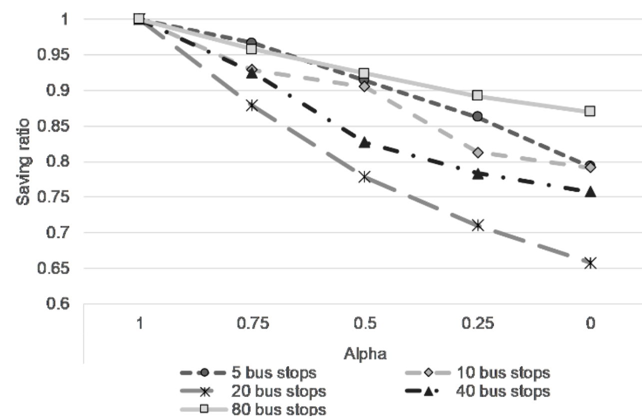

Fig. 4. Saving ratios in the instances grouped by the number of bus stops.

These results allow us to affirm that, by increasing the students walking distance by at most 20%, considerable savings can be achieved in the total distance traveled by the routes and therefore in the fuel used. This implies that the proposed fuzzy model may be meaningful for a decision-maker. For example, in the fuzzy solution of the instance Inst80 presented in Fig. 3 it can be appreciated the possibility that at the cost of increasing the walking distance from 40 ($\alpha = 1$, crisp) to 48 ($\alpha = 0$, the most relaxed), a cost-saving of more than 70% is achieved. Other intermediate values of relaxation, values of α, and cost may also be interesting.

Another analysis that can be made is to compare the archived results with solution values presented in [31]. In this comparison, the relaxation at any level brings a better solution in 18 instances. The average cost-saving in all instances is about 14%. On the other hand, if only the 18 instances with the best solutions are compared, the average cost-savings is around 25%.

Fig. 4 shows a graph in which the average savings obtained in the instances can be observed according to the number of stops, based on the different values of α (0: the greatest relaxation; 1: no relaxation, i.e., the original problem).

In Fig. 4, it can be seen that, on average, in all types of instances, some savings between 4%, when the relaxation is the smallest one (alpha=0.75), and 35%, when the relaxation is the highest one (alpha=0), are achieved. In this way, the instances with 20 stops stand out because they allow saving almost 35% when the maximum relaxation is achieved. On the contrary, the

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instances with 80 stops only allow savings of 10% with the maximum relaxation. According to this analysis, the relaxation of the walking distance seems to be less important when there are many stops due to the existence of multiple options in the same interval.

From another point of view, Fig. 5 shows a graph of the average proportional savings obtained for the different values of α in the instances, grouped by the maximum amount that a student can walk.

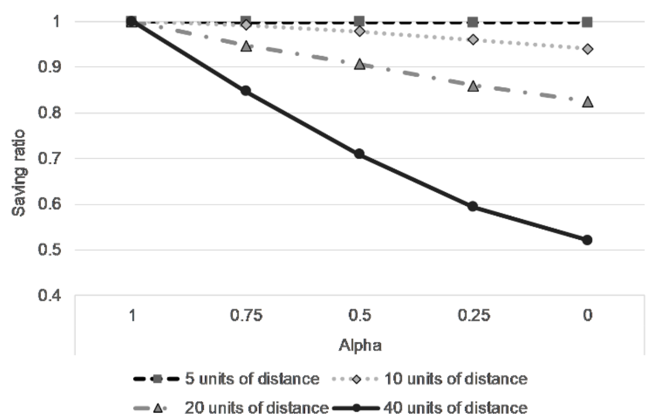

Fig. 5. Saving ratios in the instances grouped by the maximum students walking distance.

In this case, it can be seen how the greatest savings are achieved in the instances with the greatest walking distance (40), which is between 15% and 48% (greatest relaxation). On the other hand, it can be observed that in the instances with the shortest walking distance (5), practical savings are not achieved.

Likewise, the results can be analyzed, taking into account the number of students. In Fig. 6, the proportion of savings achieved for the different values of α is shown. In this case, the instances are grouped according to the number of students. Savings are achieved in all types of instances, the instances with 100 students being the ones with the most remarkable savings with values between 10% and 40% for the maximum relaxation.

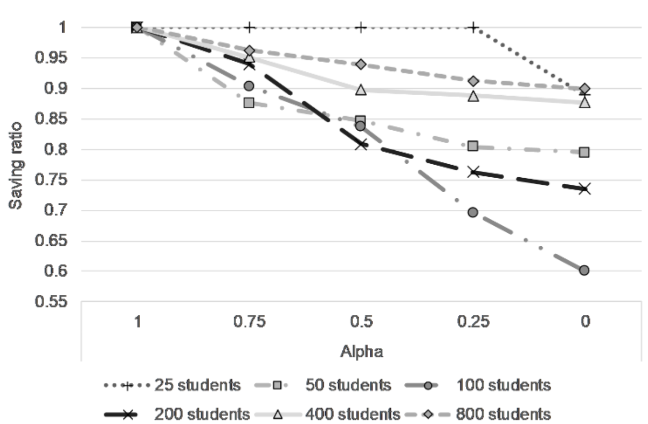

Fig. 6. Saving ratios in the instances are grouped by the number of students.

In general, the results obtained in these instances indicate that intermediate values of the number of bus stops and students and large values of walking distances in the instances tend to allow a greater reduction in the cost.

The proposed approach has the main advantage with respect to the solutions found in the literature. Our proposal allows the decision-makers to have more than one solution of interest to be considered; thus, they can evaluate different trade-offs between cost-savings and compliance with the original conditions.

In addition, from the point of view of computational cost, the parametric approach can be applied in a very efficient way. The solution found without any relaxation can be used to search for the solution with the lower allowed relaxation, and then this new solution can be used as the starting solution for the next level of relaxation and so on.

5. Conclusions

This paper introduced a fuzzy model for the SBRP that allows modeling uncertainty for the constraint associated with the student walking distance. The proposed model and its solution allow obtaining a fuzzy solution as a set of crisp solutions to the SBRP with interesting trade-offs between cost and accomplishment of the original maximum student's walking distance. A main contribution of the proposal is the possibility of offering to the decision-maker the opportunity to analyze multiple solutions with different degrees of membership to the original conditions. Likewise, the results allow us to affirm that, at the cost of slightly increasing the student's walking distance; considerable savings can be achieved in the evaluation of the objective function.

This research allows us to trace a way forward in the application of fuzzy optimization in the SBRP concerning other aspects, e.g., the capacity of buses. Another interesting aspect is to study how to set the alpha values to obtain the most interesting trade-offs between cost and relaxation.

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